

A TOPOLOGICAL VIEW ON BARYON NUMBER CONSERVATION

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We argue that the charge fractionalization in quarks has a hidden topological character related to a broken \mathbb{Z}_2 symmetry between integer-charged bare quarks and leptons. The mechanism is a tunneling process occurring in time between standard field configurations of a pure gauge form with different topological winding numbers associated with integer-charged bare quarks in the far past and future. This transition, which nonperturbatively normalizes local bare charges with a universal accumulated value, corresponds to a specific topologically nontrivial configuration of the weak gauge fields in Euclidean spacetime. The outcome is an effective topological charge equal to the ratio between baryon number and the number of fermion generations associated with baryonic matter. The observed conservation of baryon number is then related to the conservation of this bookkeeping charge on quarks. Baryon number violation may only arise through topological effects as in decays induced by electroweak instantons. However, stability of a free proton is expected.

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1. INTRODUCTION

Present experimental evidence indicates that baryons can be produced only in particle-antiparticle pairs.¹ In general, these particles are unstable and go through decay chains that end with the lightest baryon, the proton. A baryon number $B = 1$ is defined for a baryon and $B = -1$ for an antibaryon.² For all other particles the baryon number is $B = 0$. A system of many particles has a baryon number given by the corresponding algebraic sum. The selection rule which describes the observed baryon number conservation states that in any particle reaction the baryon number is the same before and after the interaction. Similar statements also apply to the conservation of the lepton number L . A necessary condition for the baryon and lepton number conservation is proton stability.

In the quark model³ all baryons are three-quark states and each quark therefore has a baryon number $B = 1/3$. The properties of quarks are correlated with the characteristics of observed particle states. So they occur in several diversities or flavors distinguished by the assignment of additive quantum numbers as in Table 1. The most unusual property of quarks is that they have fractional charges. The baryon number (B), flavor (I_z , S , C , B^* , or T), and electric charge (Q) of quarks (and observed particles) are related according to the generalized Gell-Mann–Nishijima formula⁴

$$Q = I_z + \frac{1}{2}(B + S + C + B^* + T). \quad (1)$$

The observed conservation of baryon number is then associated with conservation of the number of quarks.

During the past few decades, however, the baryon number and lepton number conservation law has been questioned by the so-called grand unified theories (GUTs) and supersymmetric grand unified theories (SGUTs),⁵ and some versions of string theory.⁶ The reason essentially comes from gauge theories which connect a charge conservation law with local gauge invariance and the existence of appropriate massless fields. No known fields are related to baryon and lepton number conservation, which are rather regarded as originating from an accidental global Abelian gauge symmetry and therefore susceptible to be violated. The main aim of GUTs is to unify all known interactions making quarks and leptons in each generation share representations of the unifying gauge group and consequently allowing direct transitions between quarks and leptons. Proton decay is one of the most striking predictions of GUTs and their supersymmetric extensions, having decay modes such as $p \rightarrow e^+ + \pi^0$ or $\bar{\nu}_\mu + K^+$ with $\Delta B = \Delta L = -1$.

TABLE 1: Quark additive quantum numbers.

Quark	u	d	s	c	b	t
B (baryon number)	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
Q (electric charge)	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$
I_z (isospin z-component)	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	0
S (strangeness)	0	0	-1	0	0	0
C (charm)	0	0	0	1	0	0
B^* (bottomness)	0	0	0	0	-1	0
T (topness)	0	0	0	0	0	1

The exact conservation law of baryon and lepton number has been dismissed anyway as a consequence of electroweak instanton effects.⁷ However, such violation effects of topological character are unobservably small. For example, while the decay $p + n + n \rightarrow e^+ + \bar{\nu}_\mu + \bar{\nu}_\tau$ with $\Delta B = \Delta L = -3$ is allowed, its probability is extremely minuscule.

On the other hand, in the most popular extension of the standard model, the minimal supersymmetric standard model (MSSM),⁸ the separate B and L conservation is usually enforced to maintain the conservation of R -parity defined as $R = (-1)^{3(B-L)+2S}$ for a particle of spin S . If this parity is conserved, the number of supersymmetric particles is then conserved. On the contrary, non-minimal models of low-energy supersymmetry with new interactions terms violating either B or L and so R -parity conservation, but avoiding proton decay, have also been discussed in the literature.⁹

It is important therefore to have new strong theoretical arguments in order to choose between these two opposite points of view for the conservation of baryon and/or lepton number and the proton stability question. In this paper we argue that at the level of perturbation theory the baryon number is conserved in any interaction coupling of local fields due to a bookkeeping charge defined in terms of a conserved effective topological charge on quarks. We also show that there is consistency with the nonperturbative topological effects that induce baryon number violations. Our arguments are based on the so-called Chern–Simons–fermion model of quarks¹⁰ recently proposed in the context of the $SU(3)_c \times SU(2)_L \times U(1)_Y$ standard model of strong and electroweak interactions.¹¹ There are two strong theoretical reasons for supposing that this new model provides the appropriate framework for understanding the physics being right beyond the standard model. These are: (a) it explains why quarks have $1/3$ electric charge relative to leptons, connecting this one-third with the number of colors; (b) it explains why quarks and leptons are so similar in their weak-interaction properties, both occurring in $SU(2)_L$ weak isospin doublets. The noticeable aspect of the approach is that these explanations are given within the scenario of the standard model itself, without need to postulate new interactions as in GUTs and preonic composite models.¹² Feature (a) is a consequence of the nontrivial topological properties of weak gauge field configurations which can nonperturbatively induce fractional charges having a topological character; the topological charge or Pontryagin index of non-Abelian fields is determined by the winding number characterizing the fields at time infinity.¹³ Thus, a quark fractional *bare* charge is seen as a result of the mixture between an integer bare charge associated with a local field (which can be derived from Noether’s theorem) and a fractional topological charge (which cannot be derived from Noether’s theorem). It is assumed that only integer bare charges are entirely associated with local fields, so that such a topological charge structure does not exist in bare leptons. This topological viewpoint for fractional charges has also been advocated by Wilczek¹⁴ on general grounds. Topological charge conservation therefore leads to an automatic conservation of baryon number at the local field level. Moreover, we find that in some sense one is proportional to the other in baryonic matter. Feature (b) is a consequence of an

electroweak \mathcal{Z}_2 symmetry between integer-charged bare quarks and leptons.

In Sec. 2 we begin by considering the charge constraints which indicate such a deeper discrete \mathcal{Z}_2 symmetry between quarks and leptons. This symmetry is then defined at a classical bare level between integer-charged bare quarks and leptons. In Sec. 3 we discuss the counterterms needed to cancel the gauge anomalies generated by the integer bare charges of quarks. We connect their nontrivial topological properties with the fractional charge required to normalize nonperturbatively to quark charge values. The resulting charges are the standard bare quark charges to begin with in the usual quantum field theory procedure. In Sec. 4 a bookkeeping charge related to baryon number is defined in terms of an effective topological charge on quarks. In Sec. 5 some concluding remarks are given.

2. INTEGER-CHARGED BARE QUARKS AND THE HIDDEN QUARK-LEPTON SYMMETRY

The starting point of our approach is the fact that the standard weak hypercharge Y of quarks and leptons are related as follows:

$$\begin{aligned} Y(u_L) &= Y(\nu_{eL}) + \frac{4}{3}, & Y(u_R) &= Y(\nu_{eR}) + \frac{4}{3}, \\ Y(d_L) &= Y(e_L) + \frac{4}{3}, & Y(d_R) &= Y(e_R) + \frac{4}{3}, \end{aligned} \tag{2}$$

and similarly for the other two generations, where the subscripts L and R denote left- and right-handed components, respectively, and the ν_R 's are introduced because of the recent experimental signatures for nonzero neutrino masses.¹ Also, since the hypercharge and electric charge operators are chosen to be related according to the form

$$Q = T_3 + \frac{1}{2}Y, \tag{3}$$

where T_i are the weak isospin generators, the electric charge of quarks and leptons become connected by

$$Q(u) = Q(\nu_e) + \frac{2}{3}, \quad Q(d) = Q(e) + \frac{2}{3}, \tag{4}$$

and similarly for the other generations. The standard assignments of weak isospin, hypercharge and electric charge are listed in Table 2. Now it should be noted that the lepton hypercharges are integer while the fractional shift for quarks, as seen from Eq. (2), is $4/3$ which depends neither on flavor nor on handedness and is conserved by electroweak interactions, i.e. one has the remarkable result that the fractional hypercharge of a quark relies just on a nonperturbative universal $4/3$ value that may have a topological character:

$$g'Y(q) = g'(m + \frac{4}{3}), \tag{5}$$

TABLE 2: Weak isospin, hypercharge and electric charge of the first generation of quarks and leptons.

Fermion	u_L	d_L	u_R	d_R	ν_{eL}	e_L	ν_{eR}	e_R
T	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0	0
T_3	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	0
Y	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{4}{3}$	$-\frac{2}{3}$	-1	-1	0	-2
Q	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	0	-1	0	-1

where m is an integer having values $0, -1, -2$ depending on quark flavor and handness, and g' is the $U(1)_Y$ coupling. Hence, to understand these patterns, it is natural to take the view that Eqs. (2) and (5) are a manifestation of a hidden electroweak \mathcal{Z}_2 symmetry between quarks and leptons as well as an indication for the fractional hypercharge of an underlying field configuration in bare quarks that may have topological attributes. This presumption was discussed in Ref. 10 within the framework of the Chern–Simons–fermion model of quarks. In general, the topological character of fractional charges has also been emphasized in Ref. 14. We remark that an alternative dynamical description would be to associate the fractional $4/3$ hypercharge with a fundamental preon and construct an appropriate composite model with new binding forces.¹² However, it is shown that the quark fractional hypercharge in Eq. (5) can be accounted for only using the topological properties of certain standard gauge field configurations, so that a preon model is not necessary.

According to the mentioned topological approach to fractional hypercharge, bare quarks may be viewed as fermions with integer bare hypercharges, referred to as prequarks, which are nonperturbatively normalized with a universal fractional piece generated by a topologically nontrivial Euclidean configuration of gauge fields. In other words, a self-consistent method that adjusts prequark hypercharges with a universal Chern–Simons contribution is pursued at the classical field theory level. Bare leptons, which have integer hypercharges, are supposed to receive no such a topological charge contribution at all. Thus, if we denote a prequark with a hat over the symbol representing the appropriate quark and the Euclidean gauge field configuration with X , bare quarks may conveniently be looked upon as “made” of the following mixtures:

$$\begin{aligned} u &= \{\hat{u}X\}, & c &= \{\hat{c}X\}, & t &= \{\hat{t}X\}, \\ d &= \{\hat{d}X\}, & s &= \{\hat{s}X\}, & b &= \{\hat{b}X\}. \end{aligned} \tag{6}$$

Of course, these structures are not authentic in the sense that gauge fields really live in Minkowski spacetime. Also, no new binding forces are involved. We argue below that in Minkowski spacetime the X -configuration is a tunneling process occurring in time by which prequarks get fractional charges. The situation is similar to the one found in instanton physics:⁷ on the one hand, an instanton is a localized pseudoparticle in Euclidean spacetime, on the other hand, it is a tunneling process in Minkowski spacetime.

Each prequark has the spin, isospin, color charge, flavor, and weak isospin of the corresponding quark; its bare weak hypercharge is the same as its lepton partner (see Eq. (2)) and its bare electric charge is defined by Eq. (3), which in turn fixes its bare baryon number through Eq. (1). Prequark charges are bare charges as prequarks are nonperturbatively dressed into bare quarks at the classical field theory level. Tables 3 and 4 list the prequark quantum numbers.

TABLE 3: Prequark additive quantum numbers.

Prequark	\hat{u}	\hat{d}	\hat{s}	\hat{c}	\hat{b}	\hat{t}
B (baryon number)	−1	−1	−1	−1	−1	−1
Q (electric charge)	0	−1	−1	0	−1	0
I_z (isospin z -component)	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	0
S (strangeness)	0	0	−1	0	0	0
C (charm)	0	0	0	1	0	0
B^* (bottomness)	0	0	0	0	−1	0
T (topness)	0	0	0	0	0	1

TABLE 4: Weak isospin, hypercharge and electric charge of the first generation of prequarks.

Prequark	\hat{u}_L	\hat{d}_L	\hat{u}_R	\hat{d}_R
T	$\frac{1}{2}$	$\frac{1}{2}$	0	0
T_3	$\frac{1}{2}$	$-\frac{1}{2}$	0	0
Y	-1	-1	0	-2
Q	0	-1	0	-1

Now regarding the bare quantum numbers associated with the X -configuration of gauge fields, from Eqs. (2), (3) and (1) we get

$$Y(X) = \frac{4}{3}, \quad Q(X) = \frac{1}{2}Y(X) = \frac{2}{3}, \quad B(X) = 2Q(X) = \frac{4}{3}. \quad (7)$$

When these fractional charges are added to the integer prequark ones, the fractional quark bare charges are obtained (see Eq. (6) and Tables 1–4). In particular, the field configuration interpolates between the prequark and quark baryon numbers. It clearly distinguishes quarks from leptons and therefore it does not allow baryon number violating fundamental couplings; i.e. the conservation law of baryon number in classical field theory is an automatic consequence of the topologically nontrivial gauge field configuration underlying bare quarks.

On the other hand, the underlying topological structure of quark fractional bare charges uncovers an electroweak \mathbb{Z}_2 symmetry between bare prequarks and leptons, which we refer to as presymmetry. This symmetry means invariance of the classical electroweak Lagrangian of prequarks and leptons under the transformation

$$\hat{u}_L^i \leftrightarrow \nu_{eL}, \quad \hat{u}_R^i \leftrightarrow \nu_{eR}, \quad \hat{d}_L^i \leftrightarrow e_L, \quad \hat{d}_R^i \leftrightarrow e_R, \quad (8)$$

and similarly for the other generations, where i denotes the color degree of freedom (see Tables 2 and 4). We observe that prequarks and leptons have the same $B - L = -1$; i.e. $B - L$ is the right fermion number to be considered under presymmetry.¹⁵ Besides, it is important to realize that presymmetry is not an *ad hoc* symmetry; as Eq. (2) shows, it underlies the relationships between quarks and leptons in the electroweak sector of the standard model.

3. COUNTERTERMS AND BARE CHARGE NORMALIZATION

We now explain how the field configuration X gets effectively involved in the local dynamics of flavors. We also prove self-consistently that quarks have fractional charges according to the quantum numbers given above. Although we follow Ref. 10, new insights into the classical dynamics of the standard gauge fields are presented.

In Minkowski spacetime the current related to the X -configuration of gauge fields is defined by the gauge-dependent form¹⁰

$$\begin{aligned} J_X^\mu &= \frac{1}{4N_{\hat{q}}} K^\mu \sum_{\hat{q}_L, \ell_L} Y + \frac{1}{16N_{\hat{q}}} L^\mu \left(\sum_{\hat{q}_L, \ell_L} Y^3 - \sum_{\hat{q}_R, \ell_R} Y^3 \right) \\ &= -\frac{1}{6} K^\mu + \frac{1}{8} L^\mu, \end{aligned} \quad (9)$$

where the sums are over all the fermion representations, $N_{\hat{q}}=36$ for three generations of prequarks, and

$$\begin{aligned} K^\mu &= \frac{g^2}{8\pi^2} \epsilon^{\mu\nu\lambda\rho} \text{tr} \left(W_\nu \partial_\lambda W_\rho - \frac{2}{3} ig W_\nu W_\lambda W_\rho \right), \\ L^\mu &= \frac{g'^2}{12\pi^2} \epsilon^{\mu\nu\lambda\rho} A_\nu \partial_\lambda A_\rho, \end{aligned} \quad (10)$$

which are the Chern–Simons classes or topological currents connected with the $SU(2)_L$ and $U(1)_Y$ gauge groups of the standard model, respectively. Their interactions are dictated by the presymmetric Lagrangian

$$\mathcal{L} = g' N_{\hat{q}} J_X^\mu A_\mu, \quad (11)$$

so that only the non-Abelian fields are topologically relevant, as expected. It is worth noting here that the X -configuration carries hypercharge, but no color and flavor. Besides, this gauge-dependent local counterterm is added to the classical Lagrangian of massless prequarks and leptons in order to remove the $U(1) \times [SU(2)]^2$ and $[U(1)]^3$ gauge anomalies generated by the integer charge of prequarks and so restore gauge invariance in a quantum field theory scenario. In fact, the $U(1)$ current for prequarks and leptons in all representations

$$\hat{J}_Y^\mu = \bar{q}_L \gamma^\mu \frac{Y}{2} \hat{q}_L + \bar{q}_R \gamma^\mu \frac{Y}{2} \hat{q}_R + \bar{\ell}_L \gamma^\mu \frac{Y}{2} \ell_L + \bar{\ell}_R \gamma^\mu \frac{Y}{2} \ell_R, \quad (12)$$

is gauge invariant but it is not conserved. Instead, due to the mentioned triangle gauge anomalies,

$$\partial_\mu \hat{J}_Y^\mu = -N_{\hat{q}} \partial_\mu J_X^\mu, \quad (13)$$

where J_X^μ is the current given in Eq. (9).

The Lagrangian in Eq. (11), however, allows us to define a new bare current:

$$J_Y^\mu = \hat{J}_Y^\mu + N_{\hat{q}} J_X^\mu, \quad (14)$$

which is conserved but it is gauge dependent. The corresponding conserved charge in Minkowski spacetime

$$Q_Y(t) = \int d^3x J_Y^o \quad (15)$$

is also gauge-dependent. Nevertheless, it has a nontrivial topological character which allows to solve all problems at once. Indeed, presymmetry has available a degree of freedom which, as shown below, can be chosen conveniently through Eq. (15) to restore gauge invariance. Of course, a fixed value breaks such a symmetry. The parameter we are referring to is the topological charge or Pontryagin index.

Actually, charge in Eq. (15) is not conserved because of the topological charge associated with gauge fields. To see how this is possible, we calculate the change in Q_Y between $t = -\infty$ and $t = +\infty$. We assume, as usual, that the region of spacetime where the energy density is nonzero is bounded.^{16,17} Therefore this region can be surrounded by a three-dimensional surface on which the field configuration becomes pure gauge, i.e.

$$W_\mu = -\frac{i}{g} (\partial_\mu U) U^{-1} \quad (16)$$

at the boundary, which can be obtained from $W_\mu = 0$ by a transformation U that takes values in the corresponding gauge group. It is convenient to use Eqs. (16), (10), (9), (14) and (15) just for the pure gauge fields. In this case we end up with

$$Q_Y(t) = \frac{N_{\hat{q}}}{6} n_W(t), \quad (17)$$

where

$$n_W(t) = \frac{1}{24\pi^2} \int d^3x \epsilon^{ijk} \text{tr}(\partial_i U U^{-1} \partial_j U U^{-1} \partial_k U U^{-1}) \quad (18)$$

is the winding number of the non-Abelian gauge transformation. This number is integer-valued if we consider a fixed time t and assume that $U(t, \mathbf{x})$ equals a direction independent constant at spatial infinity, e.g. $U \rightarrow 1$ for $|\mathbf{x}| \rightarrow \infty$. The usual argument to see this property is based on the observation that this U can be viewed as a map from the three-dimensional space with all points at infinity regarded as the same onto the three-dimensional sphere of parameters S^3 of the $SU(2)_L$ group manifold. But three-space with all points at infinity being in fact one point is topologically equivalent to a sphere S^3 in Minkowski space. Therefore U determines a map $S^3 \rightarrow S^3$. These maps can be characterized by an integer topological index which labels the homotopy class of the map. This integer is analytically given by Eq. (18). For the Abelian case, $n_W = 0$ since $A_\mu = \partial_\mu \chi$, $U = e^{ig'\chi}$, $\partial_k U U^{-1} = ig' \partial_k \chi$, and $\epsilon^{ijk} \partial_i \chi \partial_j \chi \partial_k \chi = 0$.

Thus, for the field configurations joined to prequarks which at the initial $t = -\infty$ and the final $t = +\infty$ are supposed to be of the above pure gauge form with different winding numbers, the change in charge becomes

$$\begin{aligned}\Delta Q_Y &= Q_Y(t = +\infty) - Q_Y(t = -\infty) \\ &= \frac{N_{\tilde{q}}}{6} [n_W(t = +\infty) - n_W(t = -\infty)].\end{aligned}\quad (19)$$

The difference between the integral winding numbers of the pure gauge configurations characterizing the gauge fields at the far past and future can be rewritten as the topological charge or Pontryagin index defined in Minkowski spacetime by

$$Q_T = \int d^4x \partial_\mu K^\mu = \frac{g^2}{16\pi^2} \int d^4x \text{tr}(W_{\mu\nu} \tilde{W}^{\mu\nu}), \quad (20)$$

where it is assumed that K^i decreases rapidly enough at spatial infinity and

$$\begin{aligned}W_{\mu\nu} &= \tau^a W_{\mu\nu}^a, \\ \tilde{W}_{\mu\nu} &= \frac{1}{2} \epsilon_{\mu\nu\lambda\rho} \tau^a W^{a\lambda\rho}, \\ W_{\mu\nu}^a &= \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g \epsilon^{abc} W_\mu^b W_\nu^c.\end{aligned}\quad (21)$$

It is gauge invariant, conserved and, for arbitrary fields, can take any real value. But, as shown above, for a pure gauge configuration it is integer-valued, so that using Eq. (19) we are led to

$$Q_T = n = n_W(t = +\infty) - n_W(t = -\infty), \quad (22)$$

and then

$$\Delta Q_Y = N_{\tilde{q}} \frac{n}{6}. \quad (23)$$

These equations have a special significance when the integral in Eq. (20) is analytically continued to Euclidean spacetime. The integral topological charge remains the same but it can now be associated with an Euclidean field configuration that at this point we identify with the X in Eq. (6). In this case, the meaning of Eq. (22) is that the Euclidean X -configuration interpolates in imaginary time between the real time pure-gauge configurations in the far past and future which are topologically inequivalent as they have different winding numbers. The condition is like the one established for instantons. Thus the continuous interpolation is to be considered as a tunneling process, so that a barrier must separate the initial and final gauge field configurations. We further note that if such an analytical continuation to Euclidean spacetime is ignored, nonzero topological charge then implies nonvanishing energy density at intermediate real time and so no conservation of energy. At the end of the process, there is an accumulated hypercharge on prequarks given in Eq. (23). So the charge of a prequark does not only depend on fields at a single instant of time, but also on flow of current at infinity. Specifically, Eq. (23) means that $N_{\tilde{q}}$ prequarks have to change their $Y/2$ in the same amount $n/6$; bare leptons have integer charge with no topological contribution from the gauge field configuration. For each prequark it implies the hypercharge change

$$Y_{\tilde{q}} \rightarrow Y_{\tilde{q}} + \frac{n}{3}. \quad (24)$$

Therefore the nontrivial topological properties of the X -configuration give an extra contribution to prequark hypercharge, so that the integer bare values we start with have to be shifted. Accordingly, the gauge anomalies have to be re-evaluated. It is found that anomalies are cancelled self-consistently for $n = 4$, the number of prequark flavors per generation, since now

$$\sum_{q_L, \ell_L} Y = 0, \quad \sum_{q_L, \ell_L} Y^3 - \sum_{q_R, \ell_R} Y^3 = 0. \quad (25)$$

The above nonperturbative hypercharge normalization with topological charge $n = 4$ is consistent with Eq. (5) and it means restoration of gauge invariance, breaking of the electroweak presymmetry in the Abelian sector, dressing of prequarks into fractional-charged bare quarks, and the replacement of the bare presymmetric model by the standard model. In fact, from Eqs. (25) and (9) we note that the gauge-dependent topological current J_X^μ related to the X -configuration is canceled. However, the corresponding conserved topological charge is gauge-independent and manifests itself as a universal part of prequark hypercharges according to Eq. (5). More precisely, if we consider the topological current $N_{\hat{q}} J_X^\mu$ in Eq. (14) and its associated X -configuration causing the hypercharge obtained in Eq. (23), we may define an effective prequark local current $\hat{J}_{Y,\text{eff}}^\mu$ by

$$\hat{J}_{Y,\text{eff}}^\mu = \frac{2}{3} (\bar{\hat{q}}_L \gamma^\mu \hat{q}_L + \bar{\hat{q}}_R \gamma^\mu \hat{q}_R) \quad (26)$$

to absorb their effects, namely, anomaly cancellation and induced hypercharge $4/3$ on prequarks regardless of flavor and handedness. Thus Eq. (14) takes the gauge-independent form

$$J_Y^\mu = \bar{\hat{q}}_L \gamma^\mu \frac{Y + 4/3}{2} \hat{q}_L + \bar{\hat{q}}_R \gamma^\mu \frac{Y + 4/3}{2} \hat{q}_R + \bar{\ell}_L \gamma^\mu \frac{Y}{2} \ell_L + \bar{\ell}_R \gamma^\mu \frac{Y}{2} \ell_R. \quad (27)$$

At this point, prequarks with normalized local hypercharge, which effectively includes the nonperturbative universal $4/3$ part, have to be identified with standard bare quarks. The replacement of prequarks by quarks in the strong, weak and Yukawa sectors is straightforward as they have the same color, flavor and weak isospin.

All of this is essentially done at the level of the classical Lagrangian where the current J_X^μ , which only mixes with prequark currents, is used. Again, the gauge configuration X related to this topological current in the far past and future is a pseudoparticle in Euclidean spacetime and a tunneling process in Minkowski spacetime by which a prequark hypercharge nonperturbatively changes from integer to fractional values. It is also interesting to note that as stated by the model self-consistency is the reason for the “magical” cancellation of gauge anomalies in the standard model. Moreover, the factor $1/3$ in Eq. (24) is due to the number of prequark colors (assuming same number of prequark and lepton families) introduced in Eq. (9) through $N_{\hat{q}}$, which predicts, as expected, that quarks carry $1/3$ -integral charge because they have three colors.

Our analysis shows that quarks instead of prequarks are the fermions to start with in the quantum field theory treatment. The novel news is that bare quarks have fractional charges owing to a nonperturbative universal contribution from an instanton-like classical gauge field configuration with specific topological properties (i.e. $n = 4$). And underlying this charge structure one has presymmetry as reflected in Eq. (2); an electroweak Z_2 symmetry between bare quarks and leptons which is broken by the vacuum configuration of gauge fields but it accounts for the electroweak similarities between quarks and leptons. In this sense, we recall that starting with bare fermions and exact symmetries, and ending with normalized fermions and broken or hidden symmetries is a conventional procedure in field theory which allows to understand properties of matter and interactions.

4. BOOKKEEPING CHARGE ON BARYONS

Next we remark that an effective fractional topological charge equal to $Q_T = n/N_{\hat{q}} = 1/N_c N_g = 1/9$, where N_c and N_g are the numbers of colors and generations of prequarks, respectively, can be associated with each X -configuration. Within the classical configuration of Eq. (6), it is then natural to define an effective topological charge for quarks, a bookkeeping charge which is conserved and should be related to its

fermion number. As a general rule we find that

$$Q_T = \frac{B}{N_g}. \quad (28)$$

Thus $\Delta B = N_g \Delta Q_T$ for any baryon-number violating process, i.e. only topological effects may violate baryon number.

To see the consistency of the definition for the bookkeeping charge in Eq. (28), we consider the baryon plus lepton number ($B + L$) violating processes induced nonperturbatively by electroweak instanton effects. According to Ref. 7, for three generations, one electroweak instanton characterized by a topological charge $Q_T = 1$ is associated with quark and lepton number violations given by

$$\begin{aligned} \Delta u + \Delta d &= -3, & \Delta e + \Delta \nu_e &= -1, \\ \Delta c + \Delta s &= -3, & \Delta \mu + \Delta \nu_\mu &= -1, \\ \Delta t + \Delta b &= -3, & \Delta \tau + \Delta \nu_\tau &= -1. \end{aligned} \quad (29)$$

In total, the baryon and lepton numbers are violated in three units ($\Delta B = \Delta L = -3$), matching three baryons or nine quarks (d, s , and b may mix through the Kobayashi–Maskawa matrix) and three antileptons. A decay such as $p + n + n \rightarrow \mu^+ + \bar{\nu}_e + \bar{\nu}_\tau$ is then allowed. Within the classical configuration scheme of Eq. (6), Eq. (29) implies

$$\begin{aligned} \Delta \hat{u} + \Delta \hat{d} &= -3, & \Delta X &= -3, \\ \Delta \hat{c} + \Delta \hat{s} &= -3, & \Delta X &= -3, \\ \Delta \hat{t} + \Delta \hat{b} &= -3, & \Delta X &= -3, \end{aligned} \quad (30)$$

i.e. one instanton induces a process in which nine X -configurations vanish. But the nine X 's make precisely the same topological charge of the electroweak instanton. In a sense, the definition in Eq. (28) therefore brings back topological charge conservation in quantum flavor dynamics. In Minkowski spacetime one electroweak instanton corresponds to a process which has associated the topological charge change $\Delta Q_T = 1$. It induces a process with the effective topological charge change $\Delta Q_T = -1$ associated with the vanishing of nine quarks and baryon number violation $\Delta B = -3$. We further note that consistency between the topological charge of the instanton and the effective one assigned to quarks corroborates the above value $n = 4$ of the Pontryagin index fixed by gauge anomaly constraints.

Finally, it can be seen from Eq. (28) that a baryon number violation $\Delta B = -1$ in actual experiments means an effective topological-charge change $\Delta Q_T = -1/N_g$. In particular, proton decay would imply the presence of a background gauge source with topological charge $1/N_g$.¹⁸ Stability of a free proton is expected anyway because instanton-like events cannot change topological charge by this amount. For three generations, the effective topological charge has a value $Q_T = 1/9$ for quarks and $Q_T = 1/3$ for nucleons.

5. CONCLUSION

In this paper we have considered a discrete \mathcal{Z}_2 symmetry between integer-charged bare quarks and leptons to understand the quark–lepton similarities in the electroweak sector of the standard model. New insights into the classical dynamics of the weak gauge fields which nonperturbatively give to bare quarks fractional charge relative to bare leptons have been provided. A self-consistent method to adjust bare charge values with Chern–Simons topological contributions has been pursued; it is assumed that only integer bare charges are entirely associated with local fields.

We have also presented arguments to explain the observed conservation of baryon number and predict stability of a free proton, based on the conservation of a bookkeeping charge defined in terms of an effective topological charge on quarks. They contravene GUTs and their supersymmetric extensions, where baryon

number violation couplings are introduced from the beginning, i.e. at the level of the classical Lagrangian. However, they support the MSSM and its non-minimal extensions with baryon number conservation, which then appear as the most promising approaches to low-energy supersymmetry; it is worth remarking here that while bare quarks and their supersymmetric partners have the same baryon number and underlying topological charge configuration, bare leptons and their superpartners, which carry integer charges, have zero baryon number and no such a topological charge structure at all. Of course, all of this essentially applies within the classical field theory analysis, which ends with the known standard bare fermions in a self-consistent way. These are the ones to start with in the usual quantum field theory study.

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